# The Will of the People: <br> How we vote and why it matters 

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League of Women Voters of Boulder County
Community Conversation

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## Why have elections？

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"The will of the people shall be the basis of the authority of government; this will shall be expressed in periodic and genuine elections which shall be by universal and equal suffrage and shall be held by secret vote or by equivalent free voting procedures."

- United Nations Universal Declaration of Human Rights, Article 21, December 1948

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In this case, the procedure is straightforward: Every voter votes for their preferred candidate, and the candidate with the most votes wins.

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- Plurality voting: Whichever candidate gets the most votes wins, even if their vote total is less than $50 \%$.
- Runoff elections: If no candidate wins more than $50 \%$ of the vote, a second election is held between the two candidates with the two largest vote totals in the original election.


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A: 32 \%, \quad B: 28 \%, \quad C: 40 \% .
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The initial election produces the following results:

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With a plurality vote, $C$ wins. But in a runoff election between $A$ and $C$, most of $B$ 's voters prefer $A$, and $A$ wins.

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With a plurality vote, $A$ wins with $36 \%$ of the vote. In a runoff election between $A$ and $C, C$ wins with $54 \%$ of the vote.

But a strong case could be made that candidate $B$ comes closest to representing "the will of the people."

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- General elections are almost always decided by plurality vote, and minor party candidates can easily play the role of spoiler.
- 1992 Presidential election: Clinton 43\%, Bush 38\%, Perot $19 \%$
- 2000 Presidential election in Florida: Bush $48.85 \%$, Gore $48.84 \%$, Nader $1.6 \%$


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For this reason, many attempts have been made to tweak the rules in order to improve the chances of electing more moderate candidates in primary elections, who it is hoped will fare better in the subsequent general elections.

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The blanket primary system was used in Washington, California, and Alaska until the year 2000, when the Supreme Court ruled it unconstitutional in California Democratic Party $v$. Jones because it forced political parties to endorse candidates against their will.

## Variations on the system

Nonpartisan blanket primary, a.k.a. "Jungle primary": In this system, all candidates for each office run against each other at once in the primary election, without regard to party affiliation. The top two candidates, regardless of party affiliation, advance to the general election.

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This system is currently in use for all statewide primaries except presidential primaries in Washington and California. A similar, but slightly different, system is also used in Louisiana.

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However, this can also occur when a party with minority support runs fewer candidates than the majority party and so has less vote-splitting between candidates.

## Variations on the system

For example, in Washington's 2016 election for state treasurer, the primary results were as follows:

| Candidate | Party | Vote percentage |
| :--- | :--- | :--- |
| Davidson | R | $25.09 \%$ |
| Waite | R | $23.33 \%$ |
| Liias | D | $20.36 \%$ |
| Comerford | D | $17.97 \%$ |
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Democrats received $51.57 \%$ of the primary vote but were shut out of the general election.

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Many alternate systems have been proposed over the years in order to allow voters to express more nuanced opinions.

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- For each ballot, $N$ points are given to the 1st place candidate, $N-1$ points to the 2nd place candidate, etc., down to 1 point for the last-place candidate. (Alternatively, points may range from $N-1$ down to 0 .)


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- After all points are tallied, the candidate with the most points wins.


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In our example from before, the ballots might be cast as follows. (For simplicity, assume there are exactly 100 voters.)

| Ordered preferences | Votes |
| :--- | :--- |
| $(A, B, C)$ | 36 |
| $(C, B, A)$ | 34 |
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So with this system, $B$ wins - despite coming in last place in the plurality vote!

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Today it is used in many academic and private institutions, and (with variations) even in a few political jurisdictions.

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- How to count ballots where not all candidates are ranked?
- Highly susceptible to a form of tactical manipulation called teaming or cloning.


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Example: Suppose that there are two factions, the Alphas and the Betas. The Alphas are very popular, with about $60 \%$ of the voters supporting them. The main candidates are the Alpha candidate $A$ and the Beta candidate $B$.

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Now say that the Betas decide to run a second, much less popular candidate $C$, who will receive about $10 \%$ of the Beta vote. Then the ballots might be cast as follows. (Again, assume there are 100 voters.)

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| Ordered preferences | Votes |
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| $(A, B, C)$ | 54 |
| $(A, C, B)$ | 6 |
| $(B, C, A)$ | 36 |
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| Candidate | Total points |
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| $A$ | $(54 \times 3)+(6 \times 3)+(36 \times 1)+(4 \times 1)=220$ |
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Even though $C$ takes votes away from $B$, the mere presence of $C$ in the election allows $B$ to defeat $A$.

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- Condorcet's jury theorem: If each member of a voting group is more likely than not to make a correct decision, then the probability that the highest vote of the group is the correct decision increases as the number of group members increases.
- Condorcet's paradox: With 3 or more candidates, majority preferences can become intransitive: The electorate may prefer $A$ to $B, B$ to $C$, and $C$ to $A$. (This is called a Condorcet cycle.)


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Condorcet disagreed strongly with Borda's method, because it can fail to elect the Condorcet winner (if there is one).

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- The procedure is repeated until some candidate has over $50 \%$ of the vote, and then that candidate wins the election.


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- In November 2019, New York City voted to adopt IRV for primary and special elections for several city offices.


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- In 2016, voters in Maine approved a referendum to implement (single-winner) ranked-choice voting-i.e., IRV - for statewide elections. The state Supreme Court first ruled that this system violated the state constitution, but then reversed itself in April 2018. It was used for the first time in June 2018 in Maine's primary election.


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- Maine voters also affirmed in June 2018 ( $55 \%$ to $45 \%$ ) that the state will continue using IRV, effective immediately, and it was used again in the general election for Maine's U.S. Congressional House and Senate seats in November 2018.


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- Easy to explain to voters, legislators, judges.
- Relatively resistant to tactical manipulation by strategic ranking.
- May inspire more positive campaigning, as candidates aim to become voters' second and third choices instead of attacking their opponents.


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- In 2018, Jared Golden won Maine's 2nd Congressional District election with $49.18 \%$ of votes cast.
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- Tallying must be centralized and requires ALL ballots before declaring a winner.


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The 2009 mayoral election of Burlington, VT was conducted by IRV and featured 3 main candidates:
(1) Kurt Wright (Republican)
(2) Andy Montroll (Democrat)
(3) Bob Kiss (Progressive, and the incumbent)

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Excluding minor candidates who did not affect the vote, the ballot count was as follows:

| Ranking | Votes | Ranking | Votes | Ranking | Votes |
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| (M, K, W) | 1332 | (K, M, W) | 2043 | (W, M, K) | 1513 |
| (M, W, K) | 767 | (K, W, M) | 371 | (W, K, M) | 495 |
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Second round tally: Kiss 4314, Wright 4064. So Kiss is elected.

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- 4067 voters preferred Montroll to Kiss, while 3477 preferred Kiss to Montroll.
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So Montroll was the Condorcet winner - but he was eliminated in the first round!

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Incidentally, a Borda count (assuming ties for candidates not ranked) gives
Montroll 18,425.5, Kiss 17,496, Wright 17,076.5.

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Incidentally, a Borda count (assuming ties for candidates not ranked) gives
Montroll 18,425.5, Kiss 17,496, Wright 17,076.5.
Aftermath: In 2010, Burlington repealed IRV by a vote of $52 \%$ to $48 \%$.

## Arrow's Theorem

So, if all of these methods have problems, could there possibly be a better way that takes all of this into account?

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In his 1951 Ph.D. thesis, Kenneth Arrow proved the following theorem, which helped earn him the 1972 Nobel Prize in
Economics:

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The only procedure that satisfies these conditions is dictatorship.

## Ordinal vs. Cardinal methods

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An alternative is a cardinal system, where voters give each candidate an independent rating or grade.

## Approval Voting

The simplest cardinal method is called approval voting. This system was developed in 1971 by Robert Weber as part of his Ph.D. thesis.

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Each voter votes for as many candidates as they choose, with no ranking of candidates, and the candidate with the most votes wins.

## Approval Voting

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- Approval voting is used for internal elections by the Green Party in Texas and Ohio, the Libertarian Party in Texas, and the U.S. Modern Whig Party.
- In 2018, Fargo, ND adopted approval voting for its future elections for city officials.


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- Like Borda count, tends to favor candidates with broad appeal.
- Gives minor parties greater visibility and may help them grow in support.
- Allows a voter to be more expressive by choosing how many candidates to vote for.


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## Disadvantages:

- As with IRV, it is possible that the winning candidate receives less than $50 \%$ approval, and so lacks a perceived mandate.
- Vulnerable to tactical manipulation by, e.g., bullet voting (i.e., only voting for one candidate), where it essentially reduces to plurality voting if enough voters do this.


## Score/STAR Voting

Score voting is a more nuanced cardinal method, where voters can rate each candidate on an integer scale, typically from 0 to 5 or 0 to 9 . The candidate with the highest total score wins.

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A variation called STAR (Score Then Automatic Runoff) has two steps:
(1) Use score voting to identify the top two candidates.
(2) Of these two candidates, the one who is preferred by most voters wins the election.

## Score/STAR Voting

## How does STAR Voting work?

Voters rate candidates, and ballots are counted in a two step process: Score, Then Automatic Runoff. [STAR]


## Score/STAR Voting

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A ballot measure to introduce STAR voting in Lane County, Oregon narrowly failed in November 2018 ( $47.6 \%$ yes vs. $52.4 \%$ no). Supporters hope to try again in 2020.

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## Disadvantages:

- STAR voting is a very new method, and its pros and cons are still being debated. For more info, see https://www.equal.vote/


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(2) The process limits the possible outcomes to two options only.
(3) The process encourages agents to think strategically: Once an agent has identified their preferences, they have no action at their disposal that would best defend their opinions in every situation.

## Multi-winner elections

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The $N$ candidates with the most votes (who may or may not receive votes on a majority of the ballots) are elected.

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- Like ordinary plurality voting, it is particularly vulnerable to tactical voting.
- Parties or factions are incentivized to nominate exactly the same number of candidates as the number of seats in order to avoid vote-splitting.
- It strongly disfavors minority representation: A typical result is that the most popular party or faction wins all the seats.


## Multi-winner elections

Example: Back to the Alphas and Betas! Suppose that there are 6 seats up for election, and the Alphas (with $60 \%$ voter support) run exactly 6 candidates $A_{1}, \ldots, A_{6}$.

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Assuming that all (or even most) of the Alpha voters vote for all 6 candidates, there is no strategy for the Betas to win even a single seat, regardless of how many candidates they run.

So the Betas have $40 \%$ voter support, but they gain no representation.

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The city of Lowell, MA chooses its City Council by plurality-at-large voting. In 2017, the city was sued by minority voters who claimed that this system of voting violates their voting rights by preventing minority representation on the Council.

## Multi-winner elections

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The city of Lowell, MA chooses its City Council by plurality-at-large voting. In 2017, the city was sued by minority voters who claimed that this system of voting violates their voting rights by preventing minority representation on the Council.

The community is over $40 \%$ Asian and Hispanic, but there have only been 2 people of color elected to the 9-member City Council in the last 20 years.

## Multi-winner elections

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(1) One that uses single transferable vote (STV), a multi-winner version of ranked choice voting, or
(2) A hybrid system that uses a combination of districts and at-large seats.

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An extensive analysis of both options was performed by the Metric Geometry and Gerrymandering Group at Tufts
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- Because the Asian and Latino populations are fairly dispersed throughout the city, it would be difficult to draw districts that would result in significant representation for people of color; even with optimally drawn districts, minority representation would likely be at most 1-2 seats.
- They estimated that an STV system would likely produce minority representation of 2-4 seats, even with low voter turnout.


## Multi-winner elections

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Voters opposed the STV system, with $49 \%$ in favor and $51 \%$ opposed. The director of the Cambodian Mutual Assistance Association said that the system "was too complex, even for people who vote year after year."

## Multi-winner elections

Example: Now suppose that the election for 6 seats is conducted by STV. Suppose that the Alphas and Betas each run 6 candidates, and all voters for each party rank the candidates in the same way. (Again, we assume 100 voters for simplicity.)

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The vote threshold to win a seat is $\frac{1}{6+1}=\frac{1}{7}$ of votes cast, so it takes 15 votes to win a seat.

## Multi-winner elections

Round 1:

| Ordered preferences | Votes |
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| $\left(A_{1}, A_{2}, A_{3}, A_{4}, A_{5}, A_{6}\right)$ | 60 |
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- In the first round, $A_{1}$ has the most first-place votes (60), so $A_{1}$ is elected. This election uses up 15 votes, and the 45 remaining Alpha ballots are transferred to their next-favorite candidate.


## Multi-winner elections

Round 2: $A_{1}$ already elected.

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- In the second round, $A_{2}$ has the most first-place votes (45), so $A_{2}$ is elected. This election uses up 15 votes, and the 30 remaining Alpha ballots are transferred to their next-favorite candidate.


## Multi-winner elections

Round 3: $A_{1}$ and $A_{2}$ already elected.

| Ordered preferences | Votes |
| :--- | :--- |
| $\left(A_{3}, A_{4}, A_{5}, A_{6}\right)$ | 30 |
| $\left(B_{1}, B_{2}, B_{3}, B_{4}, B_{5}, B_{6}\right)$ | 40 |

## Multi-winner elections

Round 3: $A_{1}$ and $A_{2}$ already elected.

| Ordered preferences | Votes |
| :--- | :--- |
| $\left(A_{3}, A_{4}, A_{5}, A_{6}\right)$ | 30 |
| $\left(B_{1}, B_{2}, B_{3}, B_{4}, B_{5}, B_{6}\right)$ | 40 |

- In the third round, $B_{1}$ has the most first-place votes (40), so $B_{1}$ is elected. This election uses up 15 votes, and the 25 remaining Beta ballots are transferred to their next-favorite candidate.


## Multi-winner elections

Round 4: $A_{1}, A_{2}$, and $B_{1}$ already elected.

| Ordered preferences | Votes |
| :--- | :--- |
| $\left(A_{3}, A_{4}, A_{5}, A_{6}\right)$ | 30 |
| $\left(B_{2}, B_{3}, B_{4}, B_{5}, B_{6}\right)$ | 25 |

## Multi-winner elections

Round 4: $A_{1}, A_{2}$, and $B_{1}$ already elected.

| Ordered preferences | Votes |
| :--- | :--- |
| $\left(A_{3}, A_{4}, A_{5}, A_{6}\right)$ | 30 |
| $\left(B_{2}, B_{3}, B_{4}, B_{5}, B_{6}\right)$ | 25 |

- In the fourth round, $A_{3}$ has the most first-place votes (30), so $A_{3}$ is elected. This election uses up 15 votes, and the 15 remaining Alpha ballots are transferred to their next-favorite candidate.


## Multi-winner elections

Round 5: $A_{1}, A_{2}, B_{1}$, and $A_{3}$ already elected.

| Ordered preferences | Votes |
| :--- | :--- |
| $\left(A_{4}, A_{5}, A_{6}\right)$ | 15 |
| $\left(B_{2}, B_{3}, B_{4}, B_{5}, B_{6}\right)$ | 25 |

## Multi-winner elections

Round 5: $A_{1}, A_{2}, B_{1}$, and $A_{3}$ already elected.

| Ordered preferences | Votes |
| :--- | :--- |
| $\left(A_{4}, A_{5}, A_{6}\right)$ | 15 |
| $\left(B_{2}, B_{3}, B_{4}, B_{5}, B_{6}\right)$ | 25 |

- In the fifth round, $B_{2}$ has the most first-place votes (25), so $B_{2}$ is elected. This election uses up 15 votes, and the 10 remaining Beta ballots are transferred to their next-favorite candidate.


## Multi-winner elections

Round 6: $A_{1}, A_{2}, B_{1}, A_{3}$, and $B_{2}$ already elected.

| Ordered preferences | Votes |
| :--- | :--- |
| $\left(A_{4}, A_{5}, A_{6}\right)$ | 15 |
| $\left(B_{3}, B_{4}, B_{5}, B_{6}\right)$ | 10 |

## Multi-winner elections

Round 6: $A_{1}, A_{2}, B_{1}, A_{3}$, and $B_{2}$ already elected.

| Ordered preferences | Votes |
| :--- | :--- |
| $\left(A_{4}, A_{5}, A_{6}\right)$ | 15 |
| $\left(B_{3}, B_{4}, B_{5}, B_{6}\right)$ | 10 |

- In the sixth round, $A_{4}$ has 15 votes and so wins the last seat.


## Multi-winner elections

Round 6: $A_{1}, A_{2}, B_{1}, A_{3}$, and $B_{2}$ already elected.

| Ordered preferences | Votes |
| :--- | :--- |
| $\left(A_{4}, A_{5}, A_{6}\right)$ | 15 |
| $\left(B_{3}, B_{4}, B_{5}, B_{6}\right)$ | 10 |

- In the sixth round, $A_{4}$ has 15 votes and so wins the last seat.

Final result: $A_{1}, A_{2}, B_{1}, A_{3}, B_{2}$, and $A_{4}$ are elected, in that order. So the Alphas win 4 seats and the Betas win 2 seats.

Multi-winner elections

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## Disadvantages of STV:

- Most of the same issues as IRV.
- The counting method is complicated and difficult to explain to voters.


## Where do we go from here？

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## Where do we go from here?

## WHICH VOTING SYSTEM SHOULD WE USE?

O FIRST PAST THE POST
O TOP-TWO PRIMARY
(0) LOUIIIANA PRIMARY

OO) CUMULATIVE VOTING
$\checkmark$ APPROVAL VOTING
$\checkmark$ MULTIPLE NON-TRANSFERRABLE VOTE
[3] INSTANT RUNOFF VOTNG
[1] SINGLE TRANSFERRABLE VOTE
[2] BORDA COUNT
…o.... RANGE VOTING
THE REFERENDUM WENT WELL, BUT WE CAN'T FIGURE OUT HOW TO COUNT THE BALLOTS.
(Credit: https://xkcd.com/2225/)

## Where do we go from here?

Since we can't have everything, we have to make non-mathematical choices about what factors to prioritize.

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Don't let the perfect be the enemy of the good!

## For further reading:

- Donald Saari, Chaotic Elections! A Mathematician Looks at Voting
- Jordan Ellenberg, How Not to Be Wrong: The Power of Mathematical Thinking, Chapter 17: "There is no such thing as public opinion"
- Jonathan Hodge and Richard Klima, The Mathematics of Voting and Elections: A Hands-On Approach
- George G. Szpiro, Numbers Rule: The Vexing Mathematics of Democracy, from Plato to the Present

