

Multi-Winner Approval Voting

How Does It Work? Why Should We Try It?



Vote
on
ALL!

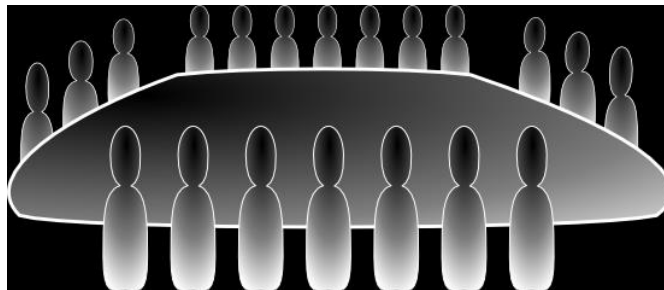


Single-Winner Contests

Appropriate for **unitary** executive offices, such as governor, treasurer, and mayor

Multi-Winner Contests

Appropriate for **multi-member** legislative or executive bodies, such as school boards, city councils and the US House of Representatives



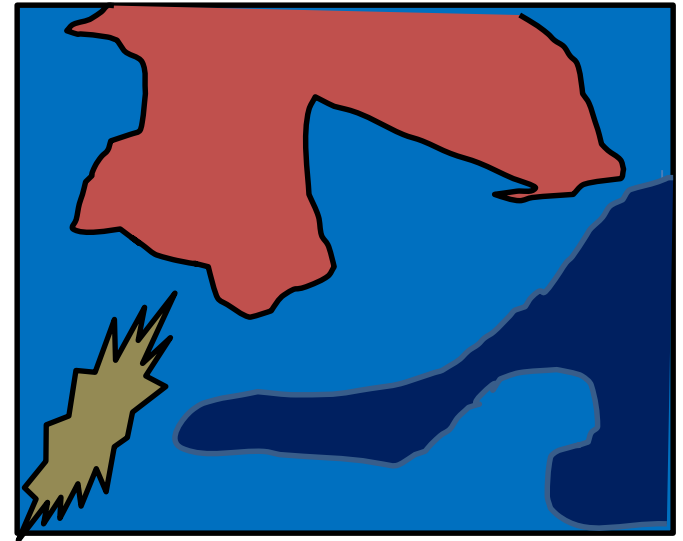
Problem: Gerrymandering!

Unfortunately, ...
many multi-member bodies
elect their members using
single-winner districts or
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districts creates safe seats in
which the voice of “minority”
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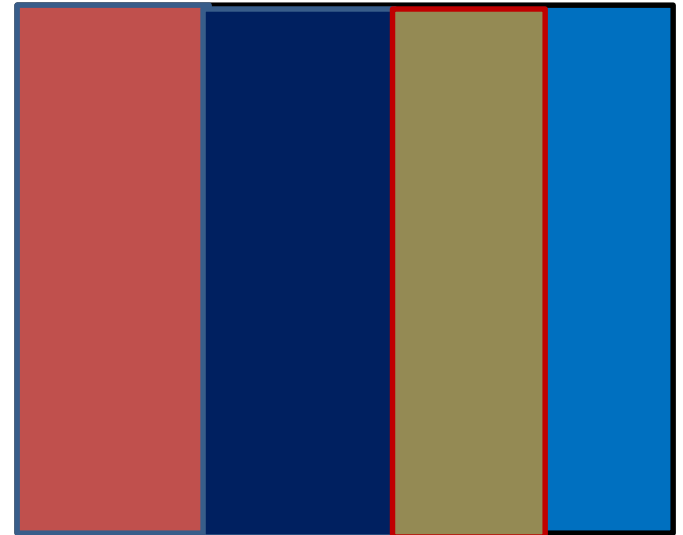
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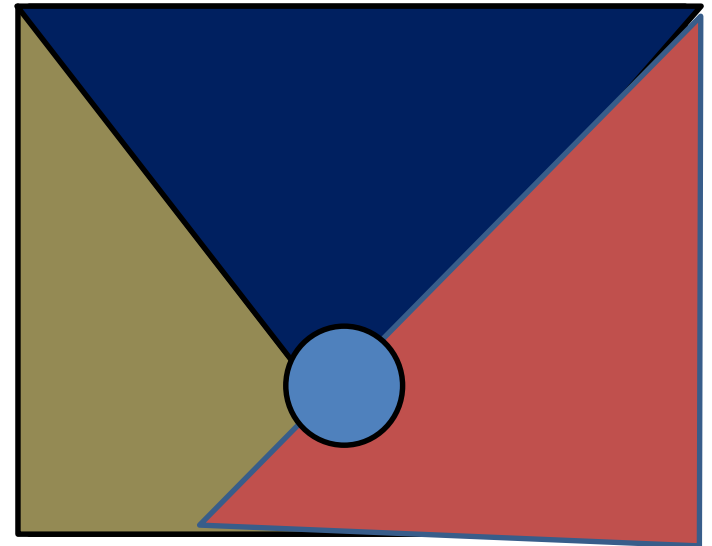
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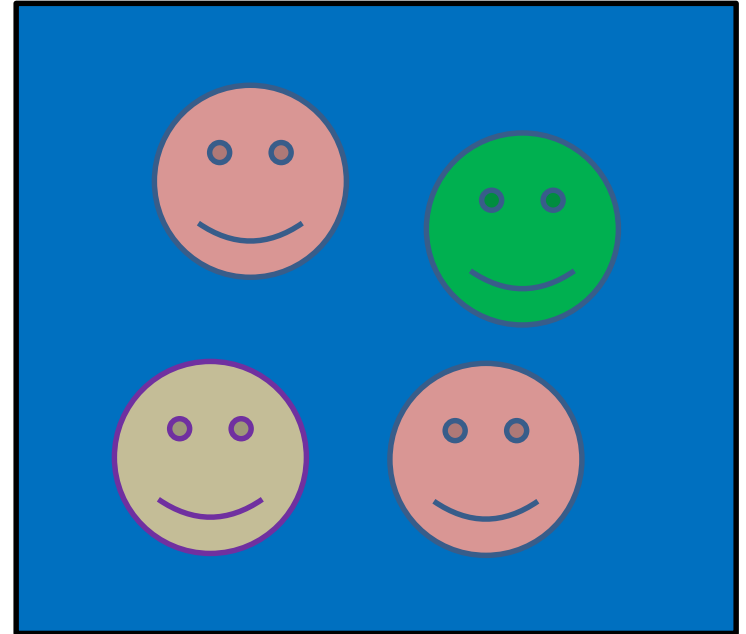
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**An Idea: Let's create multi-member districts and
hold multi-winner elections!**

Some MW Voting Methods

Plurality Block Voting

Cumulative Voting

Multi-Winner Approval Voting

Single Transferable Vote – a type of ranked voting

Other MW Voting Methods – future discussion topics

Mixed-Member Proportional Representation

Reweighted Range (or Score) Voting

GOLD Voting

Voter's Experience

s = # of seats to fill, c = # of candidates

Plurality Block Voting

→ s votes max -- 1 vote per candidate

Cumulative Voting

→ s votes max -- 1 to s votes per candidate

MW Approval Voting

→ c votes max -- 1 vote per candidate

Single Transferable Vote

→ 1 vote -- rank multiple candidates

Vote
on
ALL!
😊

Multi-Winner Ballot: Plurality Block vs Approval Block

Plurality Block Voting

Vote for (up to) three.

Voter experience: DIFFERENT
from single-winner ballot

Approval Block Voting

*Vote for every candidate you
support.*

Voter experience: IDENTICAL to
single-winner ballot and other
MW Approval voting ballots

3 candidates to be elected

o Lemon pound cake

o Lemon meringue pie

o Lemon bar

o Vanilla ice cream

o Vanilla custard

o Chocolate brownie

o Chocolate malt

o Chocolate chip cookie

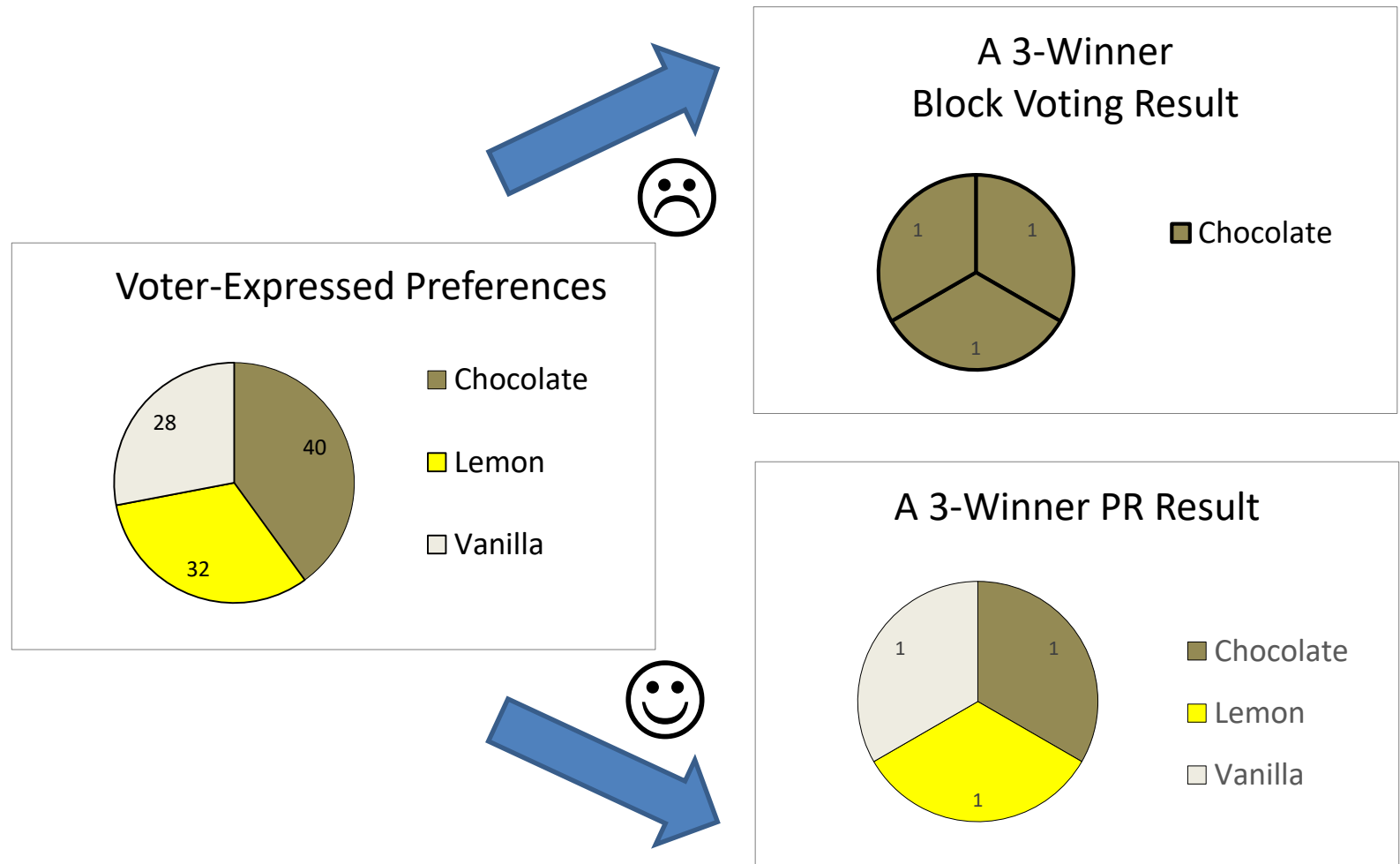
Unfortunately, ...
a **multi-winner** election is
NOT a sufficient condition
to achieve proportional
representation (PR)!

Proportional Representation

- Like-minded groups of voters are reflected proportionately in the elected body
- Avoids wasted votes
- Widely used around the world



PR or Not PR?



Multi-Winner Block Voting does NOT Promote PR

Plurality Block Voting
Vote for (up to) three.

Approval Block Voting
Vote for every candidate you support.

3 candidates to be elected

- Lemon pound cake
- Lemon meringue pie
- Lemon bar
- Vanilla ice cream
- Vanilla custard
- Chocolate brownie
- Chocolate malt
- Chocolate chip cookie

Multi-Winner Block Voting does NOT Promote PR

A **plurality** of the electorate, voting as a block, can elect their favorite candidates, even if *a majority of the electorate doesn't support those candidates.*

Sample Results:

- 3 **Lemon** candidates → votes on 32% of the ballots
- 2 **Vanilla** candidates → votes on 28% of the ballots
- 3 **Chocolate** candidates → votes on 40% of the ballots

Chocolate wins ALL the seats!

Plurality Block vs Approval Block

Similar

The voter may only give **one vote (or point)** to any single candidate.

Block voting does not promote proportional representation.

Simple to tally. Block voting methods are currently used in elections.

Different

In Approval voting voters can vote yes or no on every candidate.

Approval voting allows for more voter expressiveness.

Plurality Block voting encourages tactical voting. With extra chocolate candidates, two could split the vote and enable a lemon candidate to win. **Approval voting can avoid splitting the vote.**

VOTING METHODS Position

LWV of Colorado

*The League supports authorizing and implementing **alternatives to plurality voting** that allow people to **express their preferences more effectively**. The League supports gaining on-the-ground experience with alternative voting methods in order to ascertain whether a voting method results in outcomes that match voters' preferences as recorded on their ballots. The League supports voting methods that can improve the election experience, that encourage honest* voting rather than tactical* voting, and that consider ease of implementation.*

Considerations:

- **Some voting methods are intended for single-winner elections, others for multi-winner elections.** *It is important that the intended use of a voting method match its actual application. Multi-winner voting methods can promote **proportional representation** which fosters diversity of our elected officials.*
- *Election officials should conduct post-election analysis to evaluate the voters' usage of the voting method and the election's reflection of voters' stated preferences. There should be sufficient data transparency – for example, access to ballot records in anonymous form – for an independent analysis to be conducted by other interested groups.*

**A voting method encourages "honest" voting when it allows voters to meaningfully support all their preferred candidates, rather than leading them to either not support their favorite or "tactically" indicate a higher preference for a candidate who is not their favorite.*

MW Approval Voting Methods that **do** Promote PR

Sequential Proportional Approval Voting (SPAV) –

Thorvald Thiele → Sweden, early 1900s

- To elect n winners, conduct n **rounds**
- Reweight successful ballots in each round

Proportional Approval Voting (PAV) – Forest Simmons, 2001

- Consider all possible **sets of winners**
- Maximize utility or satisfaction of electorate

Satisfaction Approval Voting (SAV) – Steven Brams & Marc Kilgour, 2010

- Voter satisfaction = $\frac{(\# \text{ of winners approved})}{(\# \text{ of candidates approved})} \leq 1$
- **Sum voter satisfaction** for each candidate over all ballots

MW Approval Voting with 2 Seats and 5 Candidates

Candidates

(B) Lemon Bar

(M) Chocolate Malt

(V) Vanilla Custard

(C) Chocolate Chip Cookie

(P) Lemon Meringue Pie

Possible Winners

BM

MC

BV

MP

BC

VC

BP

VP

MV

CP

Note: Neither **B** alone nor **BMV** is a winning set.
However, voters may bullet vote or “overvote.”

Let's Tally a MW Approval Voting Election!

---- * ----

The polls have closed.

We have 25 marked ballots.

See your worksheet.

MW Approval Voting with 2 Seats and 5 Candidates

<u>Preferences</u>	<u># of Ballots</u>	<u>Supported</u>
Lemon	6	BP
Pie	1	P
Pastry	4	BCP
Vanilla	2	V
Liquid	3	MV
Chocolate	9	MC

Total Ballots = 25

TALLYING Approval Block Voting

AB 1/3

Count up all the votes for each candidate.

- ☐ 1st place = the highest vote-getter
- ☐ 2nd place = the next highest vote-getter

- Simple and precinct summable
- Compatible with risk-limiting audits
- Does not promote PR

TALLYING Approval Block Voting

AB 2/3

<u>5 candidates --></u> <u>Elect 2</u>	Ballot #	Lemon Bar	Chocolate Malt	Vanilla Custard	Chocolate Chip Cookie	Lemon Meringue Pie
<u>Voter Type</u>		<u>B</u>	<u>M</u>	<u>V</u>	<u>C</u>	<u>P</u>
Raw Totals		10	12	5	13	11
Block Approval						

Approval Block Voting RESULTS

AB 3/3

- Chocolate wins both seats!
 - 64% of the electorate voted for at least one Chocolate so 36% of voters did not vote for any Chocolates
 - 64% of the electorate voted for Lemons or Vanilla, including 44% supporting at least one Lemon
- Note: 16% of voters supported a Lemon candidate and a Chocolate candidate.

TALLYING Sequential Proportional Approval Voting (SPAV) SPAV 1/3

To elect n winners, conduct n rounds

□ 1st-round winner = Approval Block winner

Reweight votes on successful ballots in each round

□ 2nd-round winner = highest vote-getter using the reweighted ballots

- Uses (Jefferson) fractions to reweight
- Promotes PR

TALLYING SPAV – Round 1 SPAV 2/3

<u>5 candidates --></u> <u>Elect 2</u>	Ballot #	Lemon Bar	Chocolate Malt	Vanilla Custard	Chocolate Chip Cookie	Lemon Meringue Pie
<u>Voter Type</u>		<u>B</u>	<u>M</u>	<u>V</u>	<u>C</u>	<u>P</u>
<ballots 1 - 6>						
Pie	7					1
Pastry	8	1			1	1
Pastry	9	1			1	1
Pastry	10	1			1	1
Pastry	11	1			1	1
Vanilla	12			1		
Vanilla	13			1		
Liquid	14		1	1		
Liquid	15		1	1		
Liquid	16		1	1		
Chocolate	17		1		1	
<ballots 18 - 25>						
Raw Totals		10	12	5	13	11
Seq PAV - Rd 1		10	12	5	13	11
Seq PAV - Rd 2						

TALLYING SPAV – Round 2

SPAV 3/3

<u>5 candidates --></u> <u>Elect 2</u>	Ballot #	Lemon Bar	Chocolate Malt	Vanilla Custard	Chocolate Chip Cookie	Lemon Meringue Pie
<u>Voter Type</u>		<u>B</u>	<u>M</u>	<u>V</u>	<u>C</u>	<u>P</u>
<ballots 1 - 6>						
Pie	7					1
Pastry	8	0.5			W	0.5
Pastry	9	0.5			W	0.5
Pastry	10	0.5			W	0.5
Pastry	11	0.5			W	0.5
Vanilla	12			1		
Vanilla	13			1		
Liquid	14		1	1		
Liquid	15		1	1		
Liquid	16		1	1		
Chocolate	17		0.5		W	
<ballots 18 - 25>						
2nd-Round Totals (Reweighted)		8	7.5	5	0	9
Seq PAV - Rd 1		10	12	5	13	11
Seq PAV - Rd 2		8	7.5	5	--	9

SPAV RESULTS

SPAV 4/4

- Chocolate chip cookie and Lemon meringue pie win in the 1st and 2nd rounds respectively.
 - 80% of the ballots supported at least one winner.
 - The 3rd-place Block winner moved up to 2nd place.
- Note: The Pastry voters supported 3 candidates, including *both* the winners.

A complaint about PR is that candidates representing smaller factions can be elected.



- Small factions may gain representation by compromising and joining a coalition.
- Voting methods can be **tuned** (ahead of time) to increase the amount of support required for a candidate to be elected!

SPAV and PAV Weights – the Feature that Promotes PR

Jefferson / D'Hondt Weights

$1, 1/2, 1/3, 1/4, 1/5, 1/6, \dots$

are most commonly used, but
other weights may be used.

Choosing which weights is how
you **tune** the voting method.

How Jefferson / D'Hondt Weights Affect PR Distribution

	Surpass the Following Vote Threshold or Portion to Fill ____ Seats								
# of Seats to Fill	1 seat	2 seats	3 seats	4 seats	5 seats	6 seats	7 seats	8 seats	9 seats
2	1/3	2/3							
3	1/4	2/4	3/4						
4	1/5	2/5	3/5	4/5					
5	1/6	2/6	3/6	4/6	5/6				
6	1/7	2/7	3/7	4/7	5/7	6/7			
7	1/8	2/8	3/8	4/8	5/8	6/8	7/8		
8	1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9	
9	1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10

TALLYING PAV 1/5

Proportional Approval Voting (PAV)

Sum electorate's “utility” [U] or satisfaction for each **possible set of winners**

Emphasizes electing at least 1 supported candidate from as many ballots as possible. An SPAV and PAV principle is that voters are not twice as happy if they get 2 winners [U score < 2] as they are if they get 1 winner [U score = 1]

Winners = set with the maximum utility

TALLYING PAV 2/5

Proportional Approval Voting (PAV)

	B1	U1	B1*U1	B2	U2	B2*U2	B1*U1 + B2*U2
Possible Sets of Winners	# of ballots with 1 winner	1-winner utility		# of ballots with 2 winners	2-winner utility		Total Utility
BM	10+12=22	1	22	0	1.5	0	22
BV	10+5=15	1	15	0	1.5	0	15
BC	6+9=15	1	15	4	1.5	6	21
BP	0+1=1	1	1	10	1.5	15	
MV	9+2=11	1	11	3	1.5	4.5	
MC	3+4=7	1	7	9	1.5	13.5	

TALLYING PAV 3/5

Proportional Approval Voting (PAV)

- Uses (Jefferson) fractions to weight utility
- Computationally intensive due to summing electorate's “utility” or satisfaction for each possible set of winners
- Promotes PR



All PAV winners are equal!

No 1st- or 2nd-place winners

TALLYING PAV 4/5

Proportional Approval Voting (PAV)

Possible Sets of Winners	# of ballots with 1 winner	1-winner utility		# of ballots with 2 winners	2-winner utility		Total Utility
BM	10+12=22	1	22	0	1.5	0	22
BV	10+5=15	1	15	0	1.5	0	15
BC	6+9=15	1	15	4	1.5	6	21
BP	0+1=1	1	1	10	1.5	15	16
MV	9+2=11	1	11	3	1.5	4.5	15.5
MC	3+4=7	1	7	9	1.5	13.5	20.5
MP	12+11=23	1	23	0	1.5	0	23
VC	5+13=18	1	18	0	1.5	0	18
VP	5+11=16	1	16	0	1.5	0	16
CP	9+7=16	1	16	4	1.5	6	22

PAV RESULTS

PAV 5/5

- Chocolate malt and Lemon meringue pie are the winning combination.
 - 92% of the ballots contributed to one of the winners. Nobody voted for both winners.
 - Tie for 2nd place between L bar/ *C malt* combo and C cookie/*L pie* combo.
- Chocolate chip cookie received the most raw votes but was not elected due to PAV's emphasis on voters having at least one of their choices elected.

TALLYING SAV 1/5

Satisfaction Approval Voting (SAV)

$$\text{Individual Voter Satisfaction (IVS)} = \frac{(\# \text{ of winners approved})}{(\# \text{ of candidates approved})}$$

Sum IVS for each candidate across all ballots.

- ☐ 1st place = candidate with highest sum
- ☐ 2nd place = candidate with 2nd highest sum

TALLYING SAV 2/5

Satisfaction Approval Voting (SAV)

<u>Voter Type</u>	$1/n$	B	M	V	C	P	<i>Check Sum</i>
Lemon - BP x 6	1/2	3				3	6
Pie - P x 1	1					1	1
Pastry - BCP x 4	1/3	1.33			1.33	1.33	4
Vanilla - V x 2	1						2
Liquid - MV x 3	1/2						3
Chocolate - MC x 9	1/2						9
<i>Sums</i>	---						

TALLYING SAV 3/5

Satisfaction Approval Voting (SAV)

- Compared to SPAV and PAV, SAV is ...
 - Simpler to tally -- precinct summable
 - Not as good at promoting PR
- Weights similar to “Even and Equal” cumulative voting
- May encourage bullet voting and discourage expressing support for “extra” candidates
 - “Overvoters” cannot attain maximum voter satisfaction of 1

TALLYING SAV 4/5

Satisfaction Approval Voting (SAV)

<u>Voter Type</u>	$1/n$	B	M	V	C	P	<i>Check Sum</i>
Lemon - BP x 6	1/2	3				3	6
Pie - P x 1	1					1	1
Pastry - BCP x 4	1/3	1.33			1.33	1.33	4
Vanilla - V x 2	1			2			2
Liquid - MV x 3	1/2		1.5	1.5			3
Chocolate - MC x 9	1/2		4.5		4.5		9
<i>Sums</i>		4.33	6	3.5	5.83	5.33	

SAV RESULTS SAV 5/5

- 2 Chocolates win with highest and second highest sums of individual voter satisfaction.
- Almost the same result as Approval Block voting but order of winners is reversed because of weighting.

Compare TALLYING Results

Voting Method	Voters supporting at least 1 winner, both winners, (a winning flavor)	Lemon Bar (B)	Chocolate Malt (M)	Vanilla Custard (V)	Choc Chip Cookie (C)	Lemon Meringue Pie (P)
Approval Block	64%, 36%, (64%)		2nd place		1st place	
Seq PAV	80%, 16%, (92%)				1st round	2nd round
PAV	92%, 0%, (92%)		Co-Winner			Co-Winner
SAV	64%, 36%, (64%)		1st place		2nd place	

Transparency with Modern Voting Methods

First step in verifying outcomes, especially with modern voting methods, is to independently reproduce the calculations.

Therefore, we should ...

- Publish online voter preferences for every ballot
- Allow anyone to re-tally

Consideration: We need to preserve ballot anonymity.

Audits of Multi-Winner Contests

- Straightforward methods exist for Bayesian audits of all voting methods
- Risk-Limiting Audits for MW Approval Voting
 - SPAV is sequential; RLAs may be more complicated, like single transferable vote
 - PAV and SAV – RLAs seem to be straightforward

Outcomes of Multi-Winner Contests

Multi-winner outcomes are more nuanced

- Margins are often tighter
- Differences between outcomes are often less consequential
- Many voters are partially satisfied

MW Approval Voting – Lessons and Questions

- * Different methods → different emphases & results
- * (Re)weighting is the main tool to achieve PR and we can adjust the weights, aka “tune the method”
- * Development work is needed prior to an RLA
- * Since we can and did tally MW Approval Voting elections, we can now ask...
 - Should we use MW Approval Voting or a different multi-winner voting method?
 - If we use MW Approval Voting, which method is best for the situation under consideration?





Resources



- [LWVBC Voting Methods Team](#) – See these slides and related presentations.
- [pr voting methods section](#) of Neal McBurnett's github website
- [“Multiwinner Approval Voting: An Apportionment Approach”](#) by SJ Brams, DM Kilgour and R Pothoff, March 2017
 - Supplemental apportionment information: Chapters 9, 10, and 12 in *Numbers Rule: The Vexing Mathematics of Democracy, from Plato to the Present* by George G. Szpiro, Princeton University Press, 2010
- Wikipedia entries for SPAV, PAV and SAV